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DAMPING OF THE FREE OSCILLATIONS OF ELASTIC SYSTEMS USING GYROSCOPES†

A. V. STEPANOV

St Petersburg

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The damping of the free oscillations of an elastic system of general form to which a two-stage gyroscope is connected is considered. The coupling between the elastic system and the outer frame of the gyroscope is rigid, while the coupling between the frames is elasto-viscous. The parameters of this coupling must be chosen so that the free oscillations corresponding to the first mode of oscillations are damped as rapidly as possible. The solution of the problem is given in explicit form for an elastic system with one degree of freedom.

Free-oscillation dampers in the form of point masses connected to elastic systems by linear elasto-viscous couplings have been considered by ourselves and others in [1–4]. It was shown that if the damper is free, the damping decrement of the free oscillations cannot be greater than a certain value, which depends on the properties of the elastic system, on the orientation and on the point where the damper is attached; the maximum of the damping decrement, corresponding to the first mode of natural oscillations of the elastic system in stiffness and viscosity, was obtained: for a small damper mass it is directly proportional, and for a large mass inversely proportional, to the square root of this mass.

By introducing an additional elastic coupling between the damper and the fixed base one can arrange for the maximum possible value of the damping decrement to be grater than when it is not present. However, either the stiffness of this additional coupling and of the coupling between the elastic system and the damper must then be greater, or one of them must be negative. In this way one can generally arrange for the damping of the oscillations corresponding to the least natural frequency to be as rapid as desired [5].

One can use as an oscillation damper a two-stage gyroscope, between the frames of which there is an elasto-viscous coupling. We show below that such a damper can be more effective than a free point damper of equivalent inertial characteristics. However, only a finite value of the damping decrement can be obtained using it.

Consider a gyroscope, the outer frame of which is connected by a rigid rod OO_1 to the point O of the elastic system, while the inner frame is connected to the outer frame by a spring of stiffness c and a damper with a viscous friction coefficient h (see Fig. 1); A and B are the equatorial and axial moments of inertia of the gyroscope rotor and Ω is its angular velocity. Suppose that, in the equilibrium position, the vector $\overrightarrow{OO_1}$ is perpendicular to the plane of the outer frame and this plane is perpendicular to the plane of the inner frame. We will calculate the transfer function of this gyroscope for the case when the point O can be displaced in the direction OO_1.

If x is the displacement of this point in the given direction, θ is the angle between the plane, perpendicular to the plane of the outer frame and passing through the axis of the inner frame, and the plane of the inner frame (the nutation angle), F is the force acting from the side of point O on the outer frame (x, θ and F are functions of time) and $2L$ is the width of the outer frame, the equations of motion of the gyroscope can be converted to the form [6]

$$AL^{-1}\ddot{x} + B\Omega\dot{\theta} = LF,$$
$$A\ddot{\theta} - B\Omega L^{-1}\dot{x} = -h\dot{\theta} - c\theta \qquad (1)$$

For $x(t) = x_0e^{\lambda t}$ (where λ is a characteristic exponent) a particular solution of the second equation of system (1) will be

$$\theta = B\Omega L^{-1}\lambda(A\lambda^2 + h\lambda + c)^{-1}x_0e^{\lambda t} \qquad (2)$$

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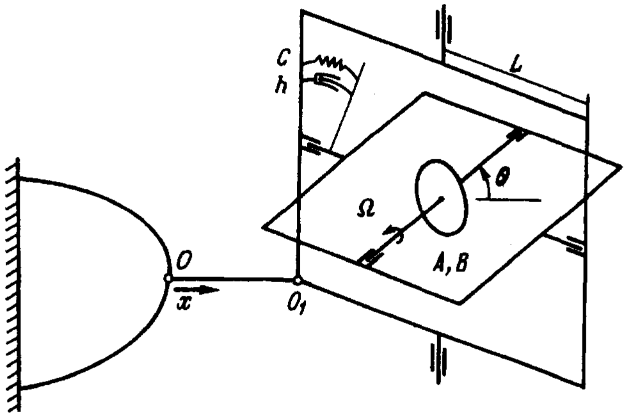

Fig. 1.

Substituting (2) into the first equation of (1) we obtain

$$F(t) = \varphi(\lambda) x_0 e^{\lambda t}$$

where $\varphi(\lambda)$ is the transfer function, which we can write in the dimensionless form

$$\varphi(\lambda) = M\Omega_1^2 f(r), \quad f(r) = \mu r^2 [1 + \beta(r^2 - zr + \sigma)^{-1}] \tag{3}$$

Here

$$\mu = \frac{A}{ML^2}, \quad z = \frac{h}{A\Omega_1}, \quad \sigma = \frac{c}{A\Omega_1^2}, \quad r = -\frac{\lambda}{\Omega_1}, \quad \beta = \left(\frac{B\Omega}{A\Omega_1}\right)^2$$

where M is the mass and Ω_1 is the lowest natural frequency of the elastic system.

The characteristic equation of the elastic system to which the gyroscopic damper is connected has the form [3]

$$(r^2 - zr + \sigma)Q(r) + \mu r^2 (r^2 - zr + \sigma + \beta)P(r) = 0 \tag{4}$$

Here

$$Q(r) = \prod_{k=1}^{S} \left[1 + \left(\frac{r}{\omega_k}\right)^2\right], \quad P(r) = \gamma(r) Q(r)$$

$$\gamma(r) = \sum_{k=1}^{S} \frac{v_k^2}{r^2 + \omega_k^2}, \quad \omega_k = \Omega_k \Omega_1^{-1}, \quad k = 1, 2, \ldots, S$$

($\gamma(r)$ is the dimensionless dynamic pliability of the elastic system at the point O in the direction OO_1. S is the number of degrees of freedom of the elastic system and $\nu_1, \nu_2, \ldots$, are the projections of the values of the eigenfunctions, normalized to its mass, at the point O onto the direction considered.)

If we now introduce the parameters and the functions

$$p = 1 + \gamma_0 \beta \mu (\gamma_0 = \gamma(0)), \quad z_\beta = p^{-1} z, \quad \sigma_\beta = p^{-1} \sigma$$

$$Q_\beta(r) = Q(r) + \mu(r^2 + \beta)P(r)$$

$$P_\beta(r) = \beta\{[r^{-2}[\gamma_0 Q(r) - P(r)] + \gamma_0 \mu P(r)\}$$

Equation (4) is equivalent to the equation

$$(r^2 - z_\beta r + \sigma_\beta) Q_\beta(r) + \mu r^2 (-z_\beta r + \sigma_\beta) P_\beta(r) = 0 \tag{5}$$

which is similar to the characteristic equation of an elastic system to which a point damper is connected by an elasto-viscous coupling [2].

It can be shown that, for any non-negative β and μ the polynomial $Q_\beta(r)$ has S pairs of pure imaginary zeros $\pm i\omega_{\beta 1}$, $\pm i\omega_{\beta 2}$, and that, for $k = 1, 2, \ldots S$

$$\lim_{r\to i\omega_{\beta k}} (r^2+\omega_{\beta k}^2)\frac{P_\beta(r)}{Q_\beta(r)} > 0$$

(we will denote this limit by $\upsilon_{\beta k}^2$); finally

$$\sum_{k=1}^{S} \frac{\upsilon_{\beta k}^2}{r^2+\omega_{\beta k}^2} = \frac{P_\beta(r)}{Q_\beta(r)}$$

For sufficiently small μ the value of $\upsilon_{\beta k}^2$ is close to $\beta(\upsilon_k\omega_k^{-1})^2$ and

$$\omega_{\beta k} = \omega_k + \frac{1}{2}(\beta\omega_k^{-1} - \omega_k)\upsilon_k^2\mu$$

When $\beta = 0$ (the gyroscope rotor is not rotating) Eq. (4) splits into two: $r^2 - zr + \sigma = 0$ and $Q(r) + \mu(r^2 + \beta)P(r) = 0$. The roots of the first correspond to decaying oscillations of the inner frame with respect to the outer frame, while those of the second correspond to free undamped oscillations of the elastic system to which a mass

$$m = AL^{-2} = \mu M \tag{6}$$

is connected by means of the rigid coupling OO_1 and which can be displaced in the direction of this coupling.

If we consider the damping at the lowest natural frequency, then, for small μ, the condition for σ_β and z_β to be optimal is the presence in Eq. (5) of a double complex root r close $i\omega_{\beta 1} \equiv i$. In that case

$$\sigma_\beta \approx i, \quad z_\beta \approx 2\upsilon_{\beta 1}\sqrt{\mu}, \quad \operatorname{Re} r \approx \upsilon_{\beta 1}\sqrt{\mu}/2$$

If $\beta > 1$, we have $\upsilon_{\beta 1} > \upsilon_1$, $\omega_{\beta 1} > \omega_1$ and using the gyroscope in question we can achieve a greater damping effect than by using a point damper of mass m, as given by (6) [1, 2].

In a system with one degree of freedom ($S = 1$, $\upsilon_1 = 1$, $\omega_1 = 1$) we have

$$\upsilon_{\beta 1}^2 = \beta, \quad \omega_{\beta 1}^2 = (1+\beta\mu)(1+\mu)^{-1}, \quad p = 1+\beta\mu$$

and the optimum values of the coupling parameters (those for which the minimum of the various damping decrements is a maximum) between the gyroscope frames are (compare with [1])

$$\sigma_\beta = \frac{\omega_{\beta 1}^2}{(1+\upsilon_{\beta 1}^2\mu)^2} = \frac{1}{(1+\mu)(1+\beta\mu)}$$

$$z_\beta = 2\upsilon_{\beta 1}\omega_{\beta 1}\left[\frac{\mu}{(1+\upsilon_{\beta 1}^2\mu)^3}\right]^{1/2} = \frac{2}{1+\beta\mu}\left(\frac{\beta\mu}{1+\mu}\right)^{1/2}$$

$$\sigma = p\sigma_\beta = \frac{1}{1+\mu}, \quad z = pz_\beta = 2\left(\frac{\beta\mu}{1+\mu}\right)^{1/2}$$

The corresponding value of the damping decrement is

$$n = \frac{1}{2}\upsilon_{\beta 1}\omega_{\beta 1}[\mu(1+\upsilon_{\beta 1}^2\mu)]^{1/2} = \frac{z}{4}$$

These formulae hold when $\beta\mu \leqslant 4$. Then the maximum possible value of the decrement is $n_+ = (1 + \mu)^{-1/2}$. For large β we have

$$n \approx \omega_{\beta 1}(\nu_{\beta 1}\sqrt{3\mu})^{-1}; \quad n \to n_+ / \sqrt{3} \quad \text{as} \quad \beta \to \infty$$

Hence, by increasing the angular velocity of the gyroscope rotor, even when its masses are small, we can achieve a finite decrement. A similar situation occurs in systems with several degrees of freedom

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